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Non-Gaussianity from Lifshitz Scalar

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A Lifshitz scalar with the dynamical critical exponent $z=3$ obtains scale-invariant, super-horizon field fluctuations without the need of an inflationary era. Since this mechanism is due to the special scaling of the Lifshitz scalar and persists in the presence of unsuppressed self-couplings, the resulting fluctuation spectrum can deviate from a Gaussian distribution. We study the non-Gaussian nature of the Lifshitz scalar's intrinsic field fluctuations, and show that primordial curvature perturbations sourced from such field fluctuations can have large non-Gaussianity of order $f_{\rm NL} = O(100)$, which will be detected by upcoming CMB observations. We compute the bispectrum and trispectrum of the fluctuations, and discuss their configurations in momentum space. In particular, the bispectrum is found to take various shapes, including the local, equilateral, and orthogonal shapes. Intriguingly, all integrals in the in-in formalism can be performed analytically.

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I. INTRODUCTION

Hořava-Lifshitz gravity [1] is attracting much attention as one of candidates for the theory of quantum gravity because of its power-counting renormalizability, which is realized by the Lifshitz scaling

$$\vec{x} \to b\vec{x}, \qquad t \to b^z t, \tag{1}$$

with the dynamical critical exponent $z \geq 3$ in the ultraviolet (UV). There are many attempts to investigate properties and implications of this theory [2, 3].

It is natural to suppose that not only gravitational fields but also other fields exhibit the same Lifshitz scaling in the UV. Even if they classically have different scalings, quantum corrections should render them to have the same scaling. A Lifshitz scalar with $z = 3$ can obtain scale-invariant, super-horizon field fluctuations even without inflation [2], thus can source the primordial curvature perturbations through mechanisms such as the curvaton scenario [4] or the modulated decay [5]. It is noteworthy that this value of z is the minimal value for which gravity is power-counting renormalizable.

In order to discern this production mechanism of the primordial perturbation from others, we need to investigate distinct features in observables such as the cosmic microwave background. In this respect, non-Gaussianity has been considered as one of the promising approaches to distinguish production mechanisms. For this reason, there are on-going efforts to detect or constrain non-Gaussian nature of the primordial perturbation [6]. Towards identification of the production mechanism by future observations, theoretical analyses of non-Gaussianity in various cosmological scenarios have been performed [7–10].

In this paper, we focus on primordial non-Gaussianity from a Lifshitz scalar and calculate its bispectrum and trispectrum. With the dynamical critical exponent $z = 3$, the scaling dimension of the Lifshitz scalar is zero and, thus, nonlinear terms in the action are unsuppressed unless forbidden by symmetry or driven to small values by renormalization. It is those nonlinear terms that we expect to produce non-Gaussianity. Even when the Lifshitz scalar's field fluctuations are linearly transformed to the curvature perturbations (which can be realized by the curvaton mechanism or/and modulated decay), it turns out that the produced bispectrum can be large enough to be observed in future observations. We find three independent cubic terms dominant in the UV, each of which gives different shape dependence of the bispectrum. Roughly speaking, they correspond to local, equilateral and orthogonal shapes, respectively.

The rest of this paper is organized as follows. In section II we review generation of scale-invariant cosmological perturbations from a Lifshitz scalar. In section III we estimate the size of non-Gaussianity and see that the nonlinear

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parameter $f_{\rm NL}$ can be as large as $O(100)$. In section IV we concretely show the momentum dependence of the bispectrum and trispectrum. Section V is devoted to a summary of this paper and discussions. In appendix A we derive the set of independent cubic and quartic terms dominant in the UV.

II. SCALE-INVARIANT POWER SPECTRUM FROM LIFSHITZ SCALAR

In this section, we review the mechanism for generation of scale-invariant cosmological perturbations from a Lifshitz scalar [2]. The action for a Lifshitz scalar Φ in Minkowski background is

$$S_\Phi = \frac{1}{2}\int dtd^3x \; \left[(\partial_t\Phi)^2 + \Phi\tilde{\mathcal{O}}\Phi + O(\Phi^3)\right], \tag{2}$$

where

$$\tilde{\mathcal{O}} = (-1)^{z+1}\frac{1}{\tilde{M}^{2(z-1)}}\Delta^z + (-1)^z\frac{s_{z-1}}{\tilde{M}^{2(z-2)}}\Delta^{z-1} + \cdots + s_1\Delta - \tilde{m}^2. \tag{3}$$

$\Delta = \delta^{ij}\partial_i\partial_j$, $\tilde{M}$ and $\tilde{m}$ are mass scales and s_n are dimensionless constants. Here, it is supposed that the time kinetic term is already canonically normalized, and thus nonlinear terms in the action indicated by $O(\Phi^3)$ do not include time derivatives. On the other hand, those nonlinear terms can include spatial derivatives. Also, the sign of the first term in the right hand side of (3) is set by requiring stability in the UV.

In the UV, the first term in $\tilde{\mathcal{O}}$ is dominant and the field Φ described by the action (3) exhibits the Lifshitz scaling (1) with

$$\Phi \to b^{(z-3)/2}\Phi. \tag{4}$$

We find that for $z = 3$, the scaling dimension of Φ is zero and thus the amplitude of quantum fluctuations of Φ is expected to be independent of the energy scale of the system of interest. This indicates that the power spectrum of quantum fluctuations of Φ in an expanding universe should be scale-invariant. Intriguingly, the minimal value of z for which Hořava-Lifshitz gravity is power-counting renormalizable is also 3. Hereafter, we consider the $z = 3$ case.

Now let us consider the Lifshitz scalar Φ, specialized to the case with $z = 3$, in a flat FRW background

$$ds^2 = -dt + a(t)^2\delta_{ij}dx^i dx^j, \tag{5}$$

to investigate generation of cosmological perturbations. We just need to replace the volume element d^3x by $d^3xa(t)^3$ and the spatial Laplacian Δ by $\Delta/a(t)^2$ in the action (2) with $z = 3$. We expand the scalar field Φ around a homogeneous v.e.v. Φ_0 as $\Phi = \Phi_0 + \phi$. Throughout this paper we consider the UV regime in which the Hubble expansion rate H is much higher than mass scales in the scalar field action. In this regime, the Hubble friction is so strong that the time dependence of the background Φ_0 is unimportant. For this reason, hereafter, we treat Φ_0 as a constant. The action for the perturbation ϕ is then written as

$$S_\phi = \frac{1}{2}\int dtd^3x\; a(t)^3\left[(\partial_t\phi)^2 + \phi\mathcal{O}\phi + O(\phi^3)\right], \tag{6}$$

where

$$\mathcal{O} = \frac{1}{M^4a(t)^6}\Delta^3 - \frac{s}{M^2a(t)^4}\Delta^2 + \frac{c_s^2}{a(t)^2}\Delta - m^2. \tag{7}$$

M and m are mass scales and s and c_s^2 are dimensionless constants. In the UV, the quadratic action for ϕ is simply

$$S_2 = \frac{1}{2}\int dtd^3x\, a(t)^3\left\{(\partial_t\phi)^2 + \frac{1}{M^4a(t)^6}\phi\Delta^3\phi\right\}. \tag{8}$$

As discussed after (4), the scaling dimension of Φ and thus ϕ is zero,

$$\phi \to b^0\phi, \tag{9}$$

and its power-spectrum should be scale-invariant. Since ϕ is scale-invariant and there is only one scale M in the UV quadratic action (8), we expect that the power-spectrum should be roughly

$$\langle\phi\phi\rangle \sim M^2. \tag{10}$$

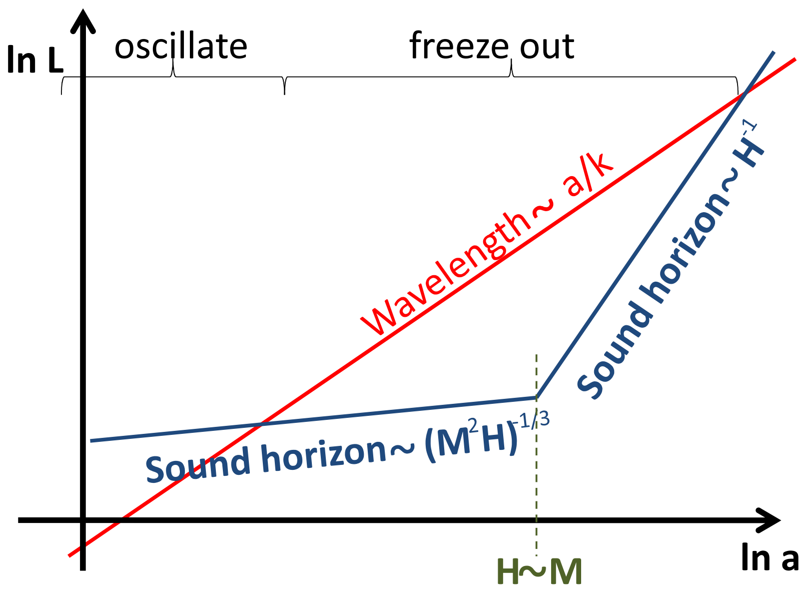

FIG. 1: A schematic picture of the evolution of cosmological perturbations for a power-law expansion $a \propto t^p$ with $1/3 < p < 1$. The physical wavelength exits the sound horizon $\sim (M^2H)^{-1/3}$ in the UV and re-enters the sound horizon $\sim H^{-1}$ in the IR. We suppose that the scale-invariant perturbations of the Lifshitz scalar are converted into those of radiation by curvaton mechanism or/and modulated decay when physical wavelengths of interest are outside the sound horizon. Thus, strictly speaking, the sound horizon in the UV is for the Lifshitz scalar and that in the IR is for radiation.

Now let us calculate the power spectrum concretely. By solving the Heisenberg equation obtained from the quadratic action (8), operator ϕ can be expanded as

$$\phi(t, \boldsymbol{x}) = \frac{1}{(2\pi)^3} \int d^3k \, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \phi_{\boldsymbol{k}}(t), \tag{11}$$

and

$$\phi_{\boldsymbol{k}}(t) = u_{\boldsymbol{k}}(t) a_{\boldsymbol{k}} + u^*_{-\boldsymbol{k}}(t) a^\dagger_{-\boldsymbol{k}}, \tag{12}$$

where

$$u_{\boldsymbol{k}}(t) = \frac{M}{2^{1/2} k^{3/2}} \exp\left(-i \frac{k^3}{M^2} \int^t \frac{dt'}{a(t')^3}\right). \tag{13}$$

The mode function $u_{\boldsymbol{k}}(t)$ is chosen so that its asymptotic behavior in the Minkowski limit ($a(t) \to$ const.) is the same as the positive-frequency mode function in Minkowski background. The vacuum state $|0\rangle$ is defined as usual by

$$a_{\boldsymbol{k}} |0\rangle = 0. \tag{14}$$

The mode function $u_{\boldsymbol{k}}(t)$ approaches a constant value in the $a(t) \to \infty$ limit if the integral $\int^{t_\infty} dt/a(t)^3$ converges, where t_∞ is the time corresponding to $a(t) \to \infty$. The power-law expansion $a(t) \propto t^p$ with $p > 1/3$ satisfies this condition. Under this condition, when the physical wavelength $a(t)/k$ becomes as long as the size of the sound horizon $\sim (M^2H)^{-1/3}$, the mode function $u_{\boldsymbol{k}}(t)$ stops oscillating and freezes out. (See in Fig.(1).) Note that the physical wavelength at sound horizon exit is super-horizon size in the UV, i.e. when $H \gg M$.

The commutation relations of operators are defined in the usual manner as

$$[\phi(\boldsymbol{x}), \pi(\boldsymbol{y})] = i\delta^3(\boldsymbol{x} - \boldsymbol{y}), \qquad [\phi(\boldsymbol{x}), \phi(\boldsymbol{y})] = [\pi(\boldsymbol{x}), \pi(\boldsymbol{y})] = 0, \tag{15}$$

where

$$\pi(\boldsymbol{x}) = \frac{\delta S_2}{\delta \partial_t \phi(\boldsymbol{x})} = a(t)^3 \partial_t \phi(\boldsymbol{x}), \tag{16}$$

and hence the operators $a_{\boldsymbol{k}}$ and $a_{\boldsymbol{k}}^\dagger$ satisfy

$$[a_{\boldsymbol{k}}, a_{\boldsymbol{k}'}^\dagger] = (2\pi)^3 \delta^3(\boldsymbol{k} - \boldsymbol{k}'), \qquad [a_{\boldsymbol{k}}, a_{\boldsymbol{k}'}] = [a_{\boldsymbol{k}}^\dagger, a_{\boldsymbol{k}'}^\dagger] = 0. \tag{17}$$

Power spectrum $\mathcal{P}_\phi$ is defined as

$$\langle 0 | \phi_{\boldsymbol{k}} \phi_{\boldsymbol{k}'} | 0 \rangle = (2\pi)^3 \delta^3(\boldsymbol{k} + \boldsymbol{k}') P_\phi, \qquad P_\phi = \frac{2\pi^2}{k^3} \mathcal{P}_\phi, \tag{18}$$

so that

$$\langle 0 | \phi^2 | 0 \rangle = \int \frac{dk}{k} \mathcal{P}_\phi. \tag{19}$$

Substituting eqs.(12) and (13) into eq.(18), we obtain

$$\mathcal{P}_\phi = \frac{M^2}{(2\pi)^2}. \tag{20}$$

In this paper, we suppose that the primordial fluctuations of the Lifshitz scalar field ϕ are almost linearly transformed to the curvature perturbations and that large non-Gaussianity is not created in the transformation itself. In scenarios like the curvaton mechanism [4] and the modulated decay [5], this assumption can be realized naturally, while violation of the assumption is also possible in these scenarios. With the assumption, the curvature perturbation ζ is related to the field fluctuation ϕ as

$$\zeta = \frac{\phi}{\mu} + O(1) \times \left(\frac{\phi}{\mu}\right)^2 + O(1) \times \left(\frac{\phi}{\mu}\right)^3 + \cdots, \tag{21}$$

where μ is an energy scale. Therefore, the power spectrum of the curvature perturbation takes over the scale-invariance as

$$\mathcal{P}_\zeta = \mu^{-2} \mathcal{P}_\phi = \frac{M^2}{(2\pi)^2 \mu^2}. \tag{22}$$

The COBE normalization [6] sets $\mathcal{P}_\zeta^{1/2} \simeq 4.9 \times 10^{-5}$, and thus

$$\frac{M}{\mu} \simeq 3.0 \times 10^{-4}. \tag{23}$$

III. ORDER ESTIMATE FOR $f_{\rm NL}$

Let us move on to discussions about non-Gaussianity. In the previous section we have seen that a free Lifshitz scalar with the dynamical critical exponent $z = 3$ can produce scale-invariant perturbations. The essential reason for this is that the scaling dimension of the scalar is zero at high energy. This implies that at high energy, some nonlinear operators could be as important as leading operators in the quadratic action and that large non-Gaussianity could be generated by those nonlinear operators. On the other hand, because of power-counting renormalizability, those nonlinear operators do not completely dominate over the quadratic terms and thus do not spoil the analysis of scaling dimensions and the scale-invariance of the power-spectrum unless the theory gets really strongly coupled. In this section, we shall present order estimates for the bispectrum of curvature perturbations and the corresponding nonlinear parameter $f_{\rm NL}$, deferring detailed calculations until the next section.

As presented in (21), throughout this paper we assume that perturbations of the Lifshitz scalar are almost linearly transformed to curvature perturbations. In our calculation of the bispectrum and trispectrum of curvature perturbations, we thus ignore nonlinear terms in the right hand side of (21) and take into account the linear term only. In particular,

$$\langle \zeta\zeta \rangle \simeq \mu^{-2} \langle \phi\phi \rangle, \quad \langle \zeta\zeta\zeta \rangle \simeq \mu^{-3} \langle \phi\phi\phi \rangle. \tag{24}$$

This treatment is justified by the fact that perturbations of the Lifshitz scalar have large non-Gaussianity. Actually, the bispectrum and trispectrum would obtain corrections from nonlinear terms in (21), but those corrections would be smaller than large contributions from the Lifshitz scalar's intrinsic non-Gaussianity. In terms of the nonlinear parameter $f_{\rm NL}$, corrections due to nonlinear terms in (21) are typically $O(1)$ but we shall see below that the scalar field's intrinsic non-Gaussianity can lead to $f_{\rm NL} = O(100)$.

We shall adopt the so called in-in formalism [7, 11] to calculate the bispectrum and trispectrum of the Lifshitz scalar. The leading contribution to the bispectrum is given by the following formula (see the next section for details)

$$\langle\phi\phi\phi\rangle = i\left\langle\left[\int dt H_3, \phi\phi\phi\right]\right\rangle, \tag{25}$$

where H_3 represents cubic terms in the interaction Hamiltonian. Dominant terms in $\int dt H_3$ are marginal ones, i.e. those terms whose scaling dimensions are zero. Actually, as shown in Appendix A, there are three (and only three) independent marginal cubic operators in the action in the UV:

$$S_3 = \int dt d^3x \, \frac{1}{M^5 a(t)^3}\left\{\alpha_1\phi^2\Delta^3\phi + \alpha_2(\Delta^2\phi)(\partial_i\phi)^2 + \alpha_3(\Delta\phi)^3\right\}, \tag{26}$$

where α_i are dimensionless parameters. (The fist term can be forbidden by the shift symmetry if one likes.) Evidently, validity of perturbative expansion (in the in-in formalism) requires α_i be smaller than unity. The corresponding cubic operators in the interaction Hamiltonian are

$$H_3(t) = -\int d^3x \, \frac{1}{M^5 a^3}\left\{\alpha_1\phi^2\Delta^3\phi + \alpha_2(\Delta^2\phi)(\partial_i\phi)^2 + \alpha_3(\Delta\phi)^3\right\}. \tag{27}$$

Each of these dominant cubic terms includes six spatial derivatives and gives zero scaling dimension to $\int dt H_3$. Combining this with the fact that the scaling dimension of ϕ is zero, we conclude that the bispectrum of ϕ given by (25) should be scale-independent, and thus

$$\langle\phi\phi\phi\rangle \sim \alpha M^3, \tag{28}$$

where α stands for the most dominant one among α_i ($i = 1, 2, 3$).

Roughly speaking, the non-linear parameter $f_{\rm NL}$ is defined so that

$$f_{\rm NL} \sim \frac{\langle\zeta\zeta\zeta\rangle}{\langle\zeta\zeta\rangle^2}. \tag{29}$$

Thus, combining this with (24), (10) and (28), we obtain

$$f_{\rm NL} \sim \alpha\left(\frac{M}{\mu}\right)^{-1} \sim 3\times 10^3\alpha. \tag{30}$$

Here, we have used the COBE normalization (23). In the next section, we obtain a more precise expression for $f_{\rm NL}$ in terms of α_i.

As already stated, validity of perturbative expansion requires that the dimensionless parameters α_i be smaller than unity. We find from the order estimate (30) that $f_{\rm NL}$ can be large, e.g. as large as $O(100)$, even if α_i are reasonably small.

IV. SHAPES OF NON-GAUSSIANITIES

We have seen that the Lifshitz scalar's non-Gaussian intrinsic fluctuations can leave observable non-Gaussianities in the sky. In this section, we compute the bispectrum and trispectrum of Lifshitz scalar fluctuations and discuss their shapes. Since a Lifshitz scalar with $z = 3$ can have all possible self-coupling terms containing six spatial derivatives (cf. (26), (53)), the resulting correlation functions can take various configurations in momentum space. We will see especially that the generated bispectrum includes local, equilateral, and orthogonal shapes. We also discuss non-Gaussianity of the primordial curvature perturbations sourced by the Lifshitz scalar's field fluctuations. By comparing with the latest observational constraints, we obtain bounds on the Lifshitz scalar's self-coupling strengths.

A. Bispectrum

We make use of the prescription of the in-in formalism (see e.g. [7, 11]) for calculating expectation values of a product $Q(t)$ of field operators at time t,

$$\langle Q(t)\rangle = \left\langle \left[\overline{T}\exp\left(i\int_{t_0}^{t} H_I(t')dt'\right)\right] Q^I(t)\left[T\exp\left(-i\int_{t_0}^{t} H_I(t')dt'\right)\right]\right\rangle, \tag{31}$$

where T denotes time-ordering, $\overline{T}$ is anti-time-ordering, $Q^I(t)$ is the product $Q(t)$ in the interaction picture, and $H_I(t)$ is the interaction part of the Hamiltonian in the interaction picture. We take the time t_0 to be at very early times when the fluctuation wavelengths are well inside the sound horizon. In the present case, introducing the time $d\tau = dt/a^3$, then the integral can be taken in terms of τ from $-\infty$ to some time after the sound horizon exit.[1]

In order to obtain the bispectrum of the Lifshitz scalar fluctuations, we expand (31) in terms of H_I and compute terms which are first order in the 3-point interaction Hamiltonian (27). We then find

$$\langle \phi_{\boldsymbol{k_1}}(t)\phi_{\boldsymbol{k_2}}(t)\phi_{\boldsymbol{k_3}}(t)\rangle = i\int_{t_0}^{t} dt' \langle [H_3(t'), \phi_{\boldsymbol{k_1}}(t)\phi_{\boldsymbol{k_2}}(t)\phi_{\boldsymbol{k_3}}(t)]\rangle \tag{32}$$

$$= -i(2\pi)^3\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2}+\boldsymbol{k_3})f(\boldsymbol{k_1},\boldsymbol{k_2}) \tag{33}$$

$$\times\, u_{\boldsymbol{k_1}}(t)u_{\boldsymbol{k_2}}(t)u_{\boldsymbol{k_3}}(t)\int_{t_0}^{t}\frac{dt'}{M^5 a(t')^3}u^*_{-\boldsymbol{k_1}}(t')u^*_{-\boldsymbol{k_2}}(t')u^*_{-\boldsymbol{k_3}}(t') + \text{c.c.} \tag{34}$$

$$= -\frac{(2\pi)^3M^3}{2^2}\frac{\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2}+\boldsymbol{k_3})f(\boldsymbol{k_1},\boldsymbol{k_2})}{(k_1k_2k_3)^3(k_1^3+k_2^3+k_3^3)}, \tag{35}$$

where we have defined

$$f(\boldsymbol{k_i},\boldsymbol{k_j}) \equiv 2\alpha_1(k_i^6+k_j^6+k_{ij}^6)+\alpha_2(k_i^6+k_j^6+k_{ij}^6-k_i^2k_j^4-k_i^4k_j^2-k_i^2k_{ij}^4-k_i^4k_{ij}^2-k_j^2k_{ij}^4-k_j^4k_{ij}^2)+6\alpha_3k_i^2k_j^2k_{ij}^2, \tag{36}$$

with $k_{ij}\equiv|\boldsymbol{k_i}+\boldsymbol{k_j}|$.

In order to study its configuration in momentum space, let us express the bispectrum (35) as

$$\langle \phi_{\boldsymbol{k_1}}(t)\phi_{\boldsymbol{k_2}}(t)\phi_{\boldsymbol{k_3}}(t)\rangle = (2\pi)^3M^3\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2}+\boldsymbol{k_3})F(k_1,k_2,k_3), \tag{37}$$

and plot $(k_1k_2k_3)^2F(k_1,k_2,k_3)$ as a function of $x_2\equiv k_2/k_1$ and $x_3\equiv k_3/k_1$. Contributions from the α_1, α_2, and α_3 terms in (36) are plotted respectively in Figures 2, 3, and 4. We have assumed $0\le x_3\le x_2\le 1$ to avoid showing the same configuration twice, and $x_2\ge 1-x_3$ further follows from the triangular inequality.

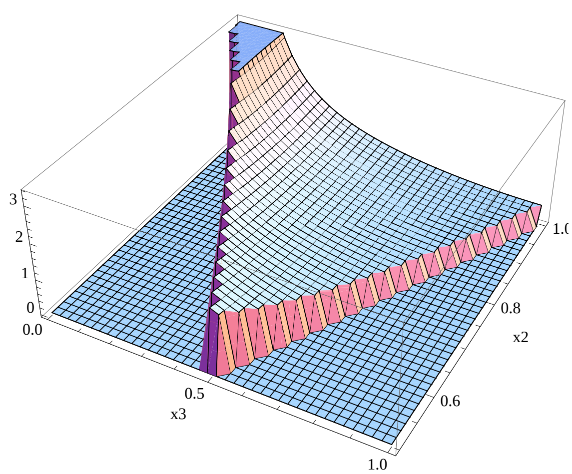

FIG. 2: The shape of the α_1 contribution to $(k_1k_2k_3)^2F$, where $\alpha_1=-1$.

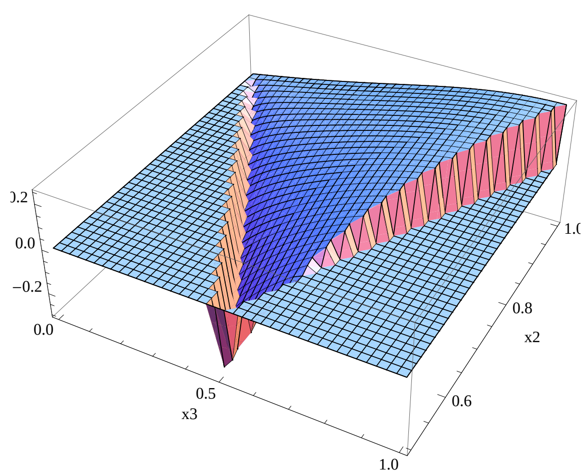

FIG. 3: The shape of the α_2 contribution to $(k_1k_2k_3)^2F$, where $\alpha_2=1$.

[1] Note that the lower limit of integration in (31) is shifted to $-\infty(1-i\epsilon)$ on the right side of Q^I, and to $-\infty(1+i\epsilon)$ on the left side, so that the oscillating exponents become exponentially decreasing. This prescription corresponds to picking up the vacuum state.

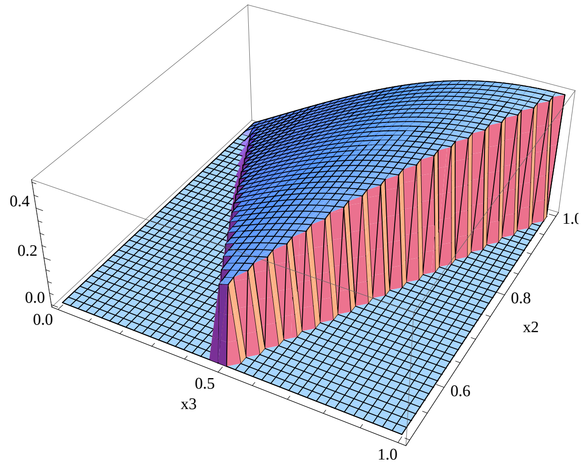

FIG. 4: The shape of the α_3 contribution to $(k_1k_2k_3)^2F$, where $\alpha_3 = -1$.

Here, one should note that only the contribution from the α_1 term blows up in the "squeezed" triangle limit, i.e. $k_3 \ll k_2 \approx k_1$, giving a shape similar to that of the local form [12]

$$F_{\rm local}(k_1,k_2,k_3) \equiv 2\left(\frac{1}{k_1^3k_2^3}+\frac{1}{k_1^3k_3^3}+\frac{1}{k_2^3k_3^3}\right). \tag{38}$$

This can be understood as follows: Among the three interaction terms in (26), only the α_1 term breaks the shift symmetry $\phi \to \phi + const$. Such a shift symmetry makes the theory indifferent to infinitely large scale fluctuations, and suppresses interactions with long wavelength modes. This is why $(k_1k_2k_3)^2F$ from the α_2 and α_3 contributions disappear in the squeezed limit.

Moreover, the shapes from the α_2 and α_3 terms respectively look similar to that of the orthogonal [13] and equilateral [14] forms which are given by

$$F_{\rm orthog.}(k_1,k_2,k_3) \equiv 6\left(-\frac{3}{k_1^3k_2^3}-\frac{3}{k_1^3k_3^3}-\frac{3}{k_2^3k_3^3}-\frac{8}{k_1^2k_2^2k_3^2}+\frac{3}{k_1k_2^2k_3^3}+(5\,\text{perm.})\right), \tag{39}$$

$$F_{\rm equil.}(k_1,k_2,k_3) \equiv 6\left(-\frac{1}{k_1^3k_2^3}-\frac{1}{k_1^3k_3^3}-\frac{1}{k_2^3k_3^3}-\frac{2}{k_1^2k_2^2k_3^2}+\frac{1}{k_1k_2^2k_3^3}+(5\,\text{perm.})\right). \tag{40}$$

The permutations act only on the last terms in the parentheses.

To give a bit more quantitative discussion on shapes, let us express (35) in terms of the above forms. Following [13, 15], we introduce a scalar product between two distributions F_1 and F_2,

$$F_1 \cdot F_2 \equiv \sum_{\boldsymbol{k_i}} \frac{F_1(k_1,k_2,k_3)F_2(k_1,k_2,k_3)}{P_{k_1}P_{k_2}P_{k_3}} \propto \int_0^1 dx_2 \int_{1-x_2}^1 dx_3\, x_2^4x_3^4 F_1(1,x_2,x_3)F_2(1,x_2,x_3), \tag{41}$$

where summation is taken over all $\boldsymbol{k_i}$'s which form a triangle, and $P_k \propto k^{-3}$ denotes the power spectrum. Then one can "expand" $F(k_1,k_2,k_3)$ of (37) in terms of (38), (39), (40) and obtain a template function:

$$F_{\rm template}(k_1,k_2,k_3) = c_{\rm NL}^{\rm local}F_{\rm local}(k_1,k_2,k_3) + c_{\rm NL}^{\rm orthog.}F_{\rm orthog.}(k_1,k_2,k_3) + c_{\rm NL}^{\rm equil.}F_{\rm equil.}(k_1,k_2,k_3), \tag{42}$$

where

$$c_{\rm NL}^{\rm local} = \frac{F\cdot F_{\rm local}}{F_{\rm local}\cdot F_{\rm local}} = -0.125\alpha_1, \tag{43}$$

$$c_{\rm NL}^{\rm orthog.} = \frac{F\cdot F_{\rm orthog.}}{F_{\rm orthog.}\cdot F_{\rm orthog.}} = 0.226\alpha_1 + 0.0186\alpha_2 - 0.00334\alpha_3, \tag{44}$$

$$c_{\rm NL}^{\rm equil.} = \frac{F\cdot F_{\rm equil.}}{F_{\rm equil.}\cdot F_{\rm equil.}} = -0.223\alpha_1 + 0.0280\alpha_2 - 0.0876\alpha_3. \tag{45}$$

α_2 and α_3 are absent in (43) since the denominator $F_{\rm local} \cdot F_{\rm local}$ blows up.[2] Here, since (38), (39), and (40) do *not* form a complete basis set, one should consider (42) as an indicator roughly telling which combination of the 3-point interaction terms in (26) gives which bispectrum shape.

Using the above template function, one can estimate the non-Gaussianity parameter $f_{\rm NL}$ of the primordial curvature perturbations. As discussed around (24), we assume that the Lifshitz scalar fluctuations are linearly converted to the curvature perturbations, i.e.

$$\zeta = \frac{\phi}{\mu}, \tag{46}$$

with some mass scale μ. For the bispectrum of Bardeen's curvature perturbations Ψ (which is related to the primordial curvature perturbations by $\zeta = \frac{5}{3}\Psi$ in the matter-dominated era)

$$\langle \Psi_{\boldsymbol{k_1}} \Psi_{\boldsymbol{k_2}} \Psi_{\boldsymbol{k_3}} \rangle = (2\pi)^3 \delta^{(3)}(\boldsymbol{k_1} + \boldsymbol{k_2} + \boldsymbol{k_3}) F_\Psi(k_1, k_2, k_3), \tag{47}$$

we define its non-Gaussianity parameter $f_{\rm NL}$ as

$$F_\Psi(k,k,k) = f_{\rm NL} \Delta_\Psi^2 \frac{6}{k^6}. \tag{48}$$

Here, Δ_Ψ is the amplitude of the power spectrum

$$\langle \Psi_{\boldsymbol{k_1}} \Psi_{\boldsymbol{k_2}} \rangle = (2\pi)^3 \delta^{(3)}(\boldsymbol{k_1} + \boldsymbol{k_2}) \frac{\Delta_\Psi}{k_1^3}, \tag{49}$$

which in the present case is

$$\Delta_\Psi = \frac{9}{50} \left(\frac{M}{\mu} \right)^2 \simeq 1.7 \times 10^{-8}. \tag{50}$$

The value on the far right hand side is set by the COBE normalization (23). Then, by substituting the template function (42) into the Lifshitz scalar fluctuations (37), we obtain

$$f^i_{\rm NL} = \frac{20}{3} \frac{\mu}{M} c^i_{\rm NL} \simeq 2.2 \times 10^4 c^i_{\rm NL}, \tag{51}$$

where i = local, orthog., equil. We can now use this relation to set constraints on the self-coupling strengths, by computing the necessary conditions for (43), (44), and (45) to satisfy the WMAP 7yr [6] constraints on the non-Gaussianity parameters: $-10 < f_{\rm NL}^{\rm local} < 74$, $-410 < f_{\rm NL}^{\rm orthog.} < 6$, and $-214 < f_{\rm NL}^{\rm equil.} < 266$ (each at 95% CL), respectively. Thus we arrive at

$$-0.03 < \alpha_1 < 0.004, \qquad -1 < \alpha_2 < 0.4, \qquad -0.5 < \alpha_3 < 0.3. \tag{52}$$

We expect that more rigorous treatments (such as using observational data to constrain combinations of different shapes, instead of individual ones) would not change the results significantly.[3]

The bounds in (52) show that observational constraints require $|\alpha_2|, |\alpha_3| = O(10^{-1})$. Note that such smallness of the coupling strengths is required anyway for validity of the perturbative expansions in terms of H_I, which we have carried out in computing the bispectrum. In other words, as α_2 or α_3 become of order unity and our procedure for computing the bispectrum breaks down, the curvature perturbations saturate the current observational limit for the orthogonal/equilateral form bispectra. The most stringent bounds are obtained for α_1, i.e. $|\alpha_1| = O(10^{-2}\text{–}10^{-3})$ where the required level of tuning depends on its sign. This is mainly due to the rather tight constraints on the local form bispectrum it produces. In order to suppress α_1, some sort of symmetry may be required in the theory. For example, as we have stated below (38), a shift symmetry for ϕ forbids such self-coupling terms producing local-type bispectra.

Here we have focused on non-Gaussianity sourced by the intrinsic fluctuations of the Lifshitz scalar, but we should remark that further non-Gaussianities in the resulting curvature perturbations can be generated through non-linear conversion processes.

---

[2] It is possible to avoid such divergences by introducing cutoffs in the integration, e.g., $x_2, x_3 \geq 0.001$ which roughly corresponds to the current observable limit in the CMB.

[3] We also note that (52) are conservative bounds, since they are necessary conditions for satisfying observational constraints. One way to obtain more strict bounds is by imposing observational constraints under an *ad hoc* assumption that two out of the three α_i's are zero. However, such procedure does not change the results significantly, except for the strong upper limit on $f_{\rm NL}^{\rm orthog.}$ tightening the upper bound on α_2 and lower bound on α_3 roughly by an order of magnitude.

B. Trispectrum

The trispectrum can be obtained in a similar manner to the previous subsection. We compute contributions from the scalar-exchange diagram (Figure 5), and from the contact-interaction diagram (Figure 6). The 4-point interaction terms of the Lifshitz scalar action are given in Appendix A as

$$S_4 = \int dt d^3x \, \frac{1}{M^6 a(t)^3} \{ \beta_1 \phi^3 \Delta^3 \phi + \beta_2 \phi^2 (\Delta\phi)(\Delta^2\phi) + \beta_3 \phi (\Delta\phi)^3 + \beta_4 \phi^2 (\Delta \partial_i \phi)^2 + \beta_5 \phi^2 (\partial_i \partial_j \partial_k \phi)^2 + \beta_6 (\partial_i \partial_j \partial_k \phi)(\partial_i \phi)(\partial_j \phi)(\partial_k \phi) \}, \quad (53)$$

where β_i are dimensionless parameters. Therefore the 4-point interaction Hamiltonian becomes

$$H_4(t) = -\int d^3x \, \frac{1}{M^6 a(t)^3} \{ \beta_1 \phi^3 \Delta^3 \phi + \beta_2 \phi^2 (\Delta\phi)(\Delta^2\phi) + \beta_3 \phi (\Delta\phi)^3 + \beta_4 \phi^2 (\Delta \partial_i \phi)^2 + \beta_5 \phi^2 (\partial_i \partial_j \partial_k \phi)^2 + \beta_6 (\partial_i \partial_j \partial_k \phi)(\partial_i \phi)(\partial_j \phi)(\partial_k \phi) \}. \quad (54)$$

Even if the 3-point interactions are suppressed, i.e. $|\alpha_i| \ll 1$, the 4-point interaction terms can produce a large trispectrum through the contact-interaction diagram.

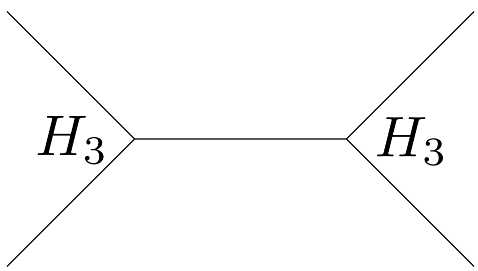

FIG. 5: The scalar-exchange diagram.

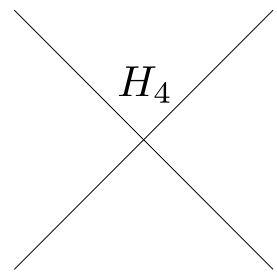

FIG. 6: The contact-interaction diagram.

The contribution from the scalar-exchange diagram are obtained as follows (Note that we only compute contributions from connected diagrams.),

$$\langle \phi_{\boldsymbol{k_1}}(t) \phi_{\boldsymbol{k_2}}(t) \phi_{\boldsymbol{k_3}}(t) \phi_{\boldsymbol{k_4}}(t) \rangle_{\text{s.e.}} = \left(-\int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \langle [H_3(t''), [H_3(t'), \phi_{\boldsymbol{k_1}}(t) \phi_{\boldsymbol{k_2}}(t) \phi_{\boldsymbol{k_3}}(t) \phi_{\boldsymbol{k_4}}(t)]] \rangle \right) \Bigg|_{\text{connected}} \quad (55)$$

$$= \frac{(2\pi)^3 M^4}{2^3} \frac{\delta^{(3)}(\boldsymbol{k_1} + \boldsymbol{k_2} + \boldsymbol{k_3} + \boldsymbol{k_4})(k_1^3 + k_2^3 + k_3^3 + k_4^3 + k_{12}^3) f(\boldsymbol{k_1}, \boldsymbol{k_2}) f(\boldsymbol{k_3}, \boldsymbol{k_4})}{(k_1 k_2 k_3 k_4 k_{12})^3 (k_1^3 + k_2^3 + k_{12}^3)(k_3^3 + k_4^3 + k_{12}^3)(k_1^3 + k_2^3 + k_3^3 + k_4^3)} \quad (56)$$

$$+ \Big(2 \text{ terms with } (1,2)(3,4) \longrightarrow (1,3)(2,4),\ (1,4)(2,3) \Big), \quad (57)$$

where $f(\boldsymbol{k_i}, \boldsymbol{k_j})$ is defined in (36). One can also compute the contribution from the contact-interaction diagram,

$$\langle \phi_{\boldsymbol{k_1}}(t) \phi_{\boldsymbol{k_2}}(t) \phi_{\boldsymbol{k_3}}(t) \phi_{\boldsymbol{k_4}}(t) \rangle_{\text{c.i.}} = \left(i \int_{t_0}^{t} dt' \langle [H_4(t'), \phi_{\boldsymbol{k_1}}(t) \phi_{\boldsymbol{k_2}}(t) \phi_{\boldsymbol{k_3}}(t) \phi_{\boldsymbol{k_4}}(t)] \rangle \right) \Bigg|_{\text{connected}} \quad (58)$$

$$= -\frac{(2\pi)^3 M^4}{2^3} \frac{\delta^{(3)}(\boldsymbol{k_1} + \boldsymbol{k_2} + \boldsymbol{k_3} + \boldsymbol{k_4}) r(\boldsymbol{k_1}, \boldsymbol{k_2}, \boldsymbol{k_3}, \boldsymbol{k_4})}{(k_1 k_2 k_3 k_4)^3 (k_1^3 + k_2^3 + k_3^3 + k_4^3)}, \quad (59)$$

where we have defined

$$
\begin{aligned}
r(\boldsymbol{k_1}, \boldsymbol{k_2}, \boldsymbol{k_3}, \boldsymbol{k_4}) \equiv& \\
&6\beta_1 \left(k_1^6 + k_2^6 + k_3^6 + k_4^6\right) \\
&+ 2\beta_2 \left(k_1^2k_2^4 + k_1^4k_2^2 + k_1^2k_3^4 + k_1^4k_3^2 + k_1^2k_4^4 + k_1^4k_4^2 + k_2^2k_3^4 + k_2^4k_3^2 + k_2^2k_4^4 + k_2^4k_4^2 + k_3^2k_4^4 + k_3^4k_4^2\right) \\
&+ 6\beta_3 \left(k_1^2k_2^2k_3^3 + k_1^2k_2^2k_4^2 + k_1^2k_3^2k_4^2 + k_2^2k_3^2k_4^2\right) \\
&+ 4\beta_4 \left(k_1^2k_2^2(\boldsymbol{k_1}\cdot\boldsymbol{k_2}) + k_1^2k_3^2(\boldsymbol{k_1}\cdot\boldsymbol{k_3}) + k_1^2k_4^2(\boldsymbol{k_1}\cdot\boldsymbol{k_4}) + k_2^2k_3^2(\boldsymbol{k_2}\cdot\boldsymbol{k_3}) + k_2^2k_4^2(\boldsymbol{k_2}\cdot\boldsymbol{k_4}) + k_3^2k_4^2(\boldsymbol{k_3}\cdot\boldsymbol{k_4})\right) \\
&+ 4\beta_5 \left((\boldsymbol{k_1}\cdot\boldsymbol{k_2})^3 + (\boldsymbol{k_1}\cdot\boldsymbol{k_3})^3 + (\boldsymbol{k_1}\cdot\boldsymbol{k_4})^3 + (\boldsymbol{k_2}\cdot\boldsymbol{k_3})^3 + (\boldsymbol{k_2}\cdot\boldsymbol{k_4})^3 + (\boldsymbol{k_3}\cdot\boldsymbol{k_4})^3\right) \\
&+ 6\beta_6 \left((\boldsymbol{k_1}\cdot\boldsymbol{k_2})(\boldsymbol{k_1}\cdot\boldsymbol{k_3})(\boldsymbol{k_1}\cdot\boldsymbol{k_4}) + (\boldsymbol{k_2}\cdot\boldsymbol{k_1})(\boldsymbol{k_2}\cdot\boldsymbol{k_3})(\boldsymbol{k_2}\cdot\boldsymbol{k_4})\right. \\
&\qquad\quad \left. + (\boldsymbol{k_3}\cdot\boldsymbol{k_1})(\boldsymbol{k_3}\cdot\boldsymbol{k_2})(\boldsymbol{k_3}\cdot\boldsymbol{k_4}) + (\boldsymbol{k_4}\cdot\boldsymbol{k_1})(\boldsymbol{k_4}\cdot\boldsymbol{k_2})(\boldsymbol{k_4}\cdot\boldsymbol{k_3})\right).
\end{aligned}
\tag{60}
$$

Expressing the trispectra as

$$
\langle \phi_{\boldsymbol{k_1}}(t)\phi_{\boldsymbol{k_2}}(t)\phi_{\boldsymbol{k_3}}(t)\phi_{\boldsymbol{k_4}}(t)\rangle = (2\pi)^3 M^4 \delta^{(4)}(\boldsymbol{k_1}+\boldsymbol{k_2}+\boldsymbol{k_3}+\boldsymbol{k_4})\mathcal{T}(\boldsymbol{k_1}, \boldsymbol{k_2}, \boldsymbol{k_3}, \boldsymbol{k_4}), \tag{61}
$$

let us plot $(k_1k_2k_3k_4)^{9/4}\mathcal{T}(\boldsymbol{k_1}, \boldsymbol{k_2}, \boldsymbol{k_3}, \boldsymbol{k_4})$ in the "equilateral" limit: $k_1 = k_2 = k_3 = k_4$. In this limit, the shape of the tetrahedron formed by $\boldsymbol{k_i}$'s depends on two independent variables, e.g., $C_2 \equiv \boldsymbol{k_1}\cdot\boldsymbol{k_2}/k_1k_2$ and $C_3 \equiv \boldsymbol{k_1}\cdot\boldsymbol{k_3}/k_1k_3$. Contributions to $\mathcal{T}_{\text{s.e.}}$ (57) from the α_1^2, α_2^2, α_3^2, $\alpha_1\alpha_2$, $\alpha_1\alpha_3$, and $\alpha_2\alpha_3$ terms are plotted respectively in Figures 7, 8, 9, 10, 11, and 12. Contributions to $\mathcal{T}_{\text{c.i.}}$ (59) from the β_1, β_5, β_6 terms are plotted in Figures 13, 14, and 15. We have omitted contributions from the β_2, β_3, β_4 terms since in the equilateral limit they become flat and are equivalent to that from β_1 up to overall constant factors. The condition $C_2 + C_3 \leq 0$ is required for $\boldsymbol{k_i}$'s to close, and we further assume $C_2 \leq C_3$ in order to avoid showing the same configuration twice.

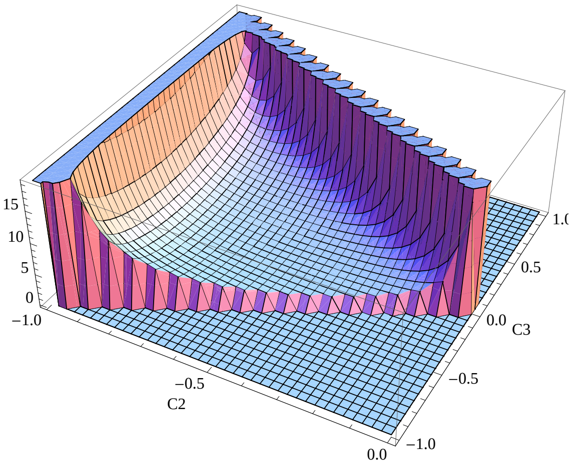

FIG. 7: The shape of the α_1^2 contribution to $(k_1k_2k_3k_4)^{9/4}\mathcal{T}_{\rm s.e.}$, where $\alpha_1^2 = 1$.

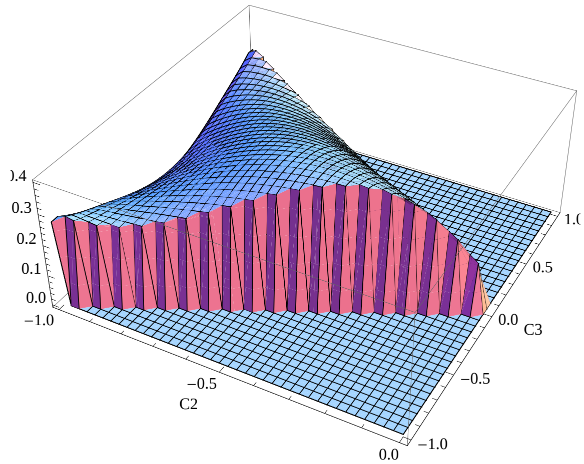

FIG. 8: The shape of the α_2^2 contribution to $(k_1k_2k_3k_4)^{9/4}\mathcal{T}_{\rm s.e.}$, where $\alpha_2^2 = 1$.

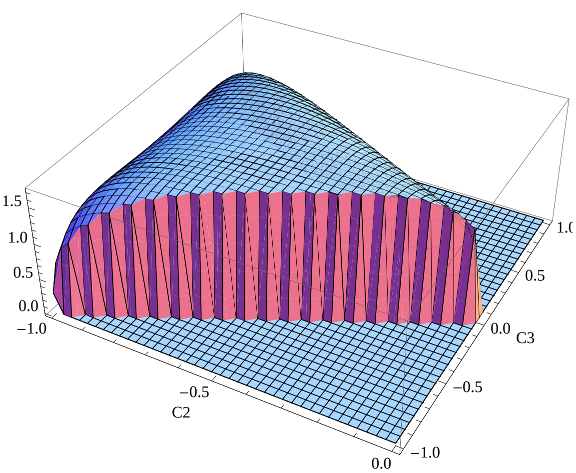

FIG. 9: The shape of the α_3^2 contribution to $(k_1k_2k_3k_4)^{9/4}\mathcal{T}_{\rm s.e.}$, where $\alpha_3^2 = 1$.

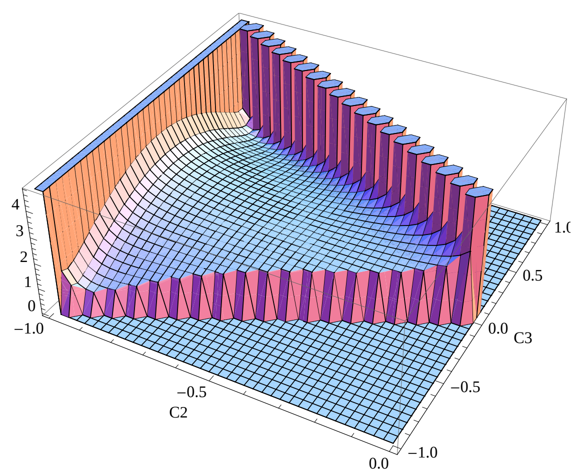

FIG. 10: The shape of the $\alpha_1\alpha_2$ contribution to $(k_1k_2k_3k_4)^{9/4}\mathcal{T}_{\rm s.e.}$, where $\alpha_1\alpha_2 = -1$.

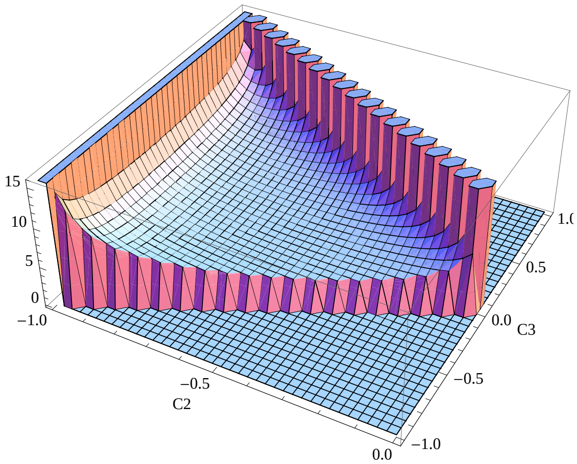

FIG. 11: The shape of the $\alpha_1\alpha_3$ contribution to $(k_1k_2k_3k_4)^{9/4}\mathcal{T}_{\mathrm{s.e.}}$, where $\alpha_1\alpha_3 = 1$.

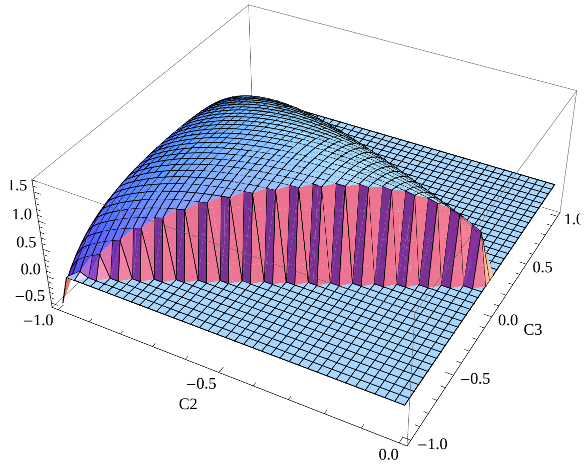

FIG. 12: The shape of the $\alpha_2\alpha_3$ contribution to $(k_1k_2k_3k_4)^{9/4}\mathcal{T}_{\mathrm{s.e.}}$, where $\alpha_2\alpha_3 = -1$.

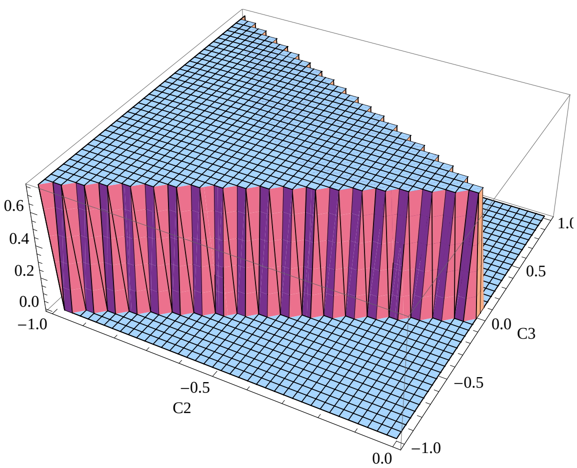

FIG. 13: The shape of the β_1 contribution to $(k_1k_2k_3k_4)^{9/4}\mathcal{T}_{\mathrm{c.i.}}$, where $\beta_1 = -1$.

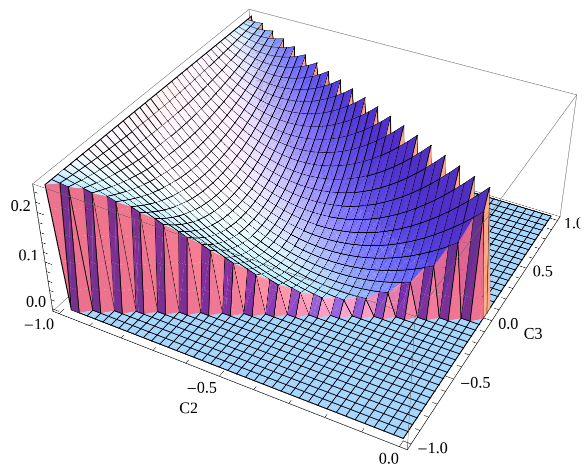

FIG. 14: The shape of the β_5 contribution to $(k_1k_2k_3k_4)^{9/4}\mathcal{T}_{\mathrm{c.i.}}$, where $\beta_5 = 1$.

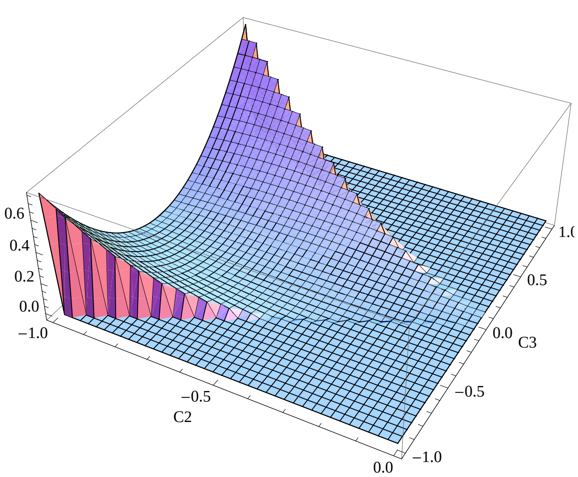

FIG. 15: The shape of the β_6 contribution to $(k_1k_2k_3k_4)^{9/4}\mathcal{T}_{\mathrm{c.i.}}$, where $\beta_6 = -1$.

V. CONCLUSIONS

In this paper, we have studied non-Gaussianity in the intrinsic fluctuations of a Lifshitz scalar which follows an anisotropic scaling with $z = 3$. Our work is based on [2], which pointed out that its special dispersion relation in the UV can lead to generation of super-horizon field perturbations. Since the scaling dimension of a Lifshitz scalar with $z = 3$ is zero, the resulting field perturbations become scale-invariant whether or not the scalar's self-couplings are small. This leads to our main point that curvature perturbations generated from such field fluctuations necessarily leave large non-Gaussianity in the sky, unless the field's self-couplings are forbidden by some symmetry, or the field exhibits some sort of asymptotic freedom. This is to be contrasted with perturbations generated through cosmic inflation, where largely non-Gaussian intrinsic fluctuations are in most cases incompatible with scale-invariance.

The Lifshitz scalar's self-coupling terms containing spatial derivatives produce non-Gaussianities with various configurations in momentum space. In particular, the bispectrum of the field fluctuations includes shapes which are similar to that of the local, equilateral, and orthogonal forms. (However, we emphasize that the local, equilateral, and orthogonal shapes do not form a complete basis set for the bispectrum obtained in this paper. We also note that the results of the effective field theory approach in [16] do not apply to our case, where Lorentz symmetry is explicitly broken, and non-Gaussianity is sourced by marginal terms in the action.) Upon computing the correlation functions, we have carried out expansions in terms of the interaction Hamiltonian. Within the domain of applicability of such perturbative expansion, i.e. the self-couplings less than unity, we have seen that the Lifshitz scalar's field fluctuations can lead to significant non-Gaussianity in the primordial curvature perturbations. In particular, when curvature perturbations are sourced linearly from the field fluctuations as in (46), their bispectrum saturates the current observational limit for the orthogonal and equilateral forms, as the self-couplings α_2 and α_3 in (26) approach unity. Since naively there is no reason for such self-couplings to be suppressed, we can expect large non-Gaussianity to be produced from Lifshitz scalar fluctuations, which may be detected by upcoming CMB observations. On the other hand, for the local-type bispectrum, observational constraints require α_1 to be as small as $O(10^{-2}–10^{-3})$ (the level of tuning depends on α_1's sign). However, as we have remarked, such self-couplings sourcing local-type non-Gaussianity can be forbidden by a shift symmetry.

The field fluctuations generated in the mechanism of [2] obtain a scale-invariant spectrum. However, when one takes into account the renormalization-group flow of the parameters of the theory (e.g. M in (8)), the spectrum may become tilted. A time-dependent background value $\Phi_0(t)$ may also give rise to similar effects. How strong the tilt becomes, as well as the scale-dependence of the non-Gaussianity, remains to be understood. While in this paper we have studied fluctuations of scalar fields, the scalar graviton which can show up in Hořava-Lifshitz gravity may also obtain fluctuations in a similar manner. It would be interesting to investigate the possibility that such scalar graviton generates the primordial curvature perturbations. (Ref. [17] works in this direction. See [18] for some issues related to the scalar graviton, including non-perturbative continuity of the limit in which general relativity is supposed to be recovered. See also [19] for a recent attempt to eliminate the scalar graviton from the theory.) Furthermore, when considering cosmic inflation in Hořava-Lifshitz gravity, due to the field fluctuations freezing-out at the time of sound horizon $(M^2H)^{-1/3}$ exit, the well-known relations in slow-roll inflation between various cosmological observables and the slow-roll parameters are expected to be modified. Aspects of cosmic inflation in Hořava-Lifshitz gravity are also worthy of study in details.

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Appendix A: Nonlinear terms in the UV action

In this appendix we construct nonlinear, marginal terms in the action in the UV, specializing to the $z = 3$ case. As mentioned after (3), those nonlinear terms do not include time derivatives but can include spatial derivatives. We demand that each term in the action can include only up to six spatial derivatives. This treatment is justified since with $z = 3$, terms with more than six spatial derivatives would be non-renormalizable and thus would not be generated by quantum corrections. In this case the most important terms in the UV are marginal ones, i.e. those

with six spatial derivatives. In the following we write down all independent combinations of cubic and quartic terms which are marginal in the UV [4].

1. Cubic terms

In general we can write down fourteen cubic terms with six spatial derivatives as follows,

$$\begin{array}{llll} A_1 \equiv \left(\Delta^3\phi\right)\phi^2, & A_2 \equiv \left(\Delta^2\partial_i\phi\right)(\partial_i\phi)\,\phi, & A_3 \equiv \left(\Delta^2\phi\right)(\Delta\phi)\,, & A_4 \equiv (\Delta\partial_i\partial_j\phi)(\partial_i\partial_j\phi)\,\phi, \\ A_5 \equiv (\Delta\partial_i\phi)(\Delta\partial_i\phi)\,\phi, & A_6 \equiv (\partial_i\partial_j\partial_k\phi)(\partial_i\partial_j\partial_k\phi)\,\phi, & A_7 \equiv \left(\Delta^2\phi\right)(\partial_i\phi)(\partial_i\phi)\,, & A_8 \equiv (\Delta\partial_i\partial_j\phi)(\partial_i\phi)(\partial_j\phi)\,, \\ A_9 \equiv (\Delta\partial_i\phi)(\Delta\phi)(\partial_i\phi)\,, & A_{10} \equiv (\Delta\partial_i\phi)(\partial_i\partial_j\phi)(\partial_j\phi)\,, & A_{11} \equiv (\partial_i\partial_j\partial_k\phi)(\partial_i\partial_j\phi)(\partial_k\phi)\,, & \\ A_{12} \equiv (\Delta\phi)(\Delta\phi)(\Delta\phi)\,, & A_{13} \equiv (\Delta\phi)(\partial_i\partial_j\phi)(\partial_i\partial_j\phi)\,, & A_{14} \equiv (\partial_i\partial_j\phi)(\partial_j\partial_k\phi)(\partial_k\partial_i\phi)\,. & \end{array} \tag{A1}$$

Some linear combinations of these terms turn out to be total derivatives. Thus, not all of these fourteen terms lead to independent terms in the action, provided that surface terms are dropped. Apparently, there are thirteen total derivatives with six spatial derivatives as follows,

$$\begin{array}{ll} \partial_i\left[\left(\Delta^2\partial_i\phi\right)\phi^2\right] = A_1 + 2A_2, & \partial_i\left[\left(\Delta^2\phi\right)(\partial_i\phi)\,\phi\right] = A_2 + A_3 + A_7, \\ \partial_i\left[(\Delta\partial_i\partial_j\phi)(\partial_j\phi)\,\phi\right] = A_2 + A_4 + A_8, & \partial_i\left[(\Delta\partial_i\phi)(\Delta\phi)\,\phi\right] = A_3 + A_5 + A_9, \\ \partial_i\left[(\Delta\partial_j\phi)(\partial_i\partial_j\phi)\,\phi\right] = A_4 + A_5 + A_{10}, & \partial_i\left[(\partial_i\partial_j\partial_k\phi)(\partial_j\partial_k\phi)\,\phi\right] = A_4 + A_6 + A_{11}, \\ \partial_i\left[(\Delta\partial_i\phi)(\partial_j\phi)(\partial_j\phi)\right] = A_7 + 2A_{10}, & \partial_i\left[(\Delta\partial_j\phi)(\partial_i\phi)(\partial_j\phi)\right] = A_8 + A_9 + A_{10}, \\ \partial_i\left[(\partial_i\partial_j\partial_k\phi)(\partial_j\phi)(\partial_k\phi)\right] = A_8 + 2A_{11}, & \partial_i\left[(\Delta\phi)(\Delta\phi)(\partial_i\phi)\right] = 2A_9 + A_{12}, \\ \partial_i\left[(\Delta\phi)(\partial_i\partial_j\phi)(\partial_j\phi)\right] = A_9 + A_{10} + A_{13}, & \partial_i\left[(\partial_i\partial_j\phi)(\partial_j\partial_k\phi)(\partial_k\phi)\right] = A_{10} + A_{11} + A_{14}, \\ \partial_i\left[(\partial_j\partial_k\phi)(\partial_j\partial_k\phi)(\partial_i\phi)\right] = 2A_{11} + A_{13}. & \end{array} \tag{A2}$$

Actually, two of the combinations in (A2) are not independent and (A2) include only eleven independent combinations. As a result, we can represent arbitrary marginal cubic terms in the action as linear combinations of three among fourteen in (A1).

Concretely, in this paper we shall use the following three terms.

$$S_3 = \int dt d^3x\,\frac{1}{M^5 a(t)^3}\left\{\alpha_1\phi^2\Delta^3\phi + \alpha_2(\Delta^2\phi)(\partial_i\phi)^2 + \alpha_3(\Delta\phi)^3\right\}. \tag{A3}$$

An advantage of this representation is that contributions from these three terms roughly correspond to local, equilateral and orthogonal shape of bispectrum, respectively.

If we impose the shift symmetry for ϕ then the number of independent interaction terms is reduced. With the shift symmetry, we have to consider only the last eight terms in (A1) and only the last seven combinations in (A2). One of the last seven combinations in (A2) is not independent. As a result, the marginal cubic terms in the action with the shift symmetry can be written as (A3) with $\alpha_1 = 0$.

2. Quartic terms

Similarly, we can write down twenty quartic terms with six spatial derivatives as follows,

$$\begin{array}{lll} B_1 \equiv \left(\Delta^3\phi\right)\phi^3, & B_2 \equiv \left(\Delta^2\partial_i\phi\right)(\partial_i\phi)\,\phi^2, & B_3 \equiv \left(\Delta^2\phi\right)(\Delta\phi)\,\phi^2, \\ B_4 \equiv (\Delta\partial_i\partial_j\phi)(\partial_i\partial_j\phi)\,\phi^2, & B_5 \equiv \left(\Delta^2\phi\right)(\partial_i\phi)(\partial_i\phi)\,\phi, & B_6 \equiv (\Delta\partial_i\partial_j\phi)(\partial_i\phi)(\partial_j\phi)\,\phi, \\ B_7 \equiv (\Delta\partial_i\phi)(\Delta\partial_i\phi)\,\phi^2, & B_8 \equiv (\partial_i\partial_j\partial_k\phi)(\partial_i\partial_j\partial_k\phi)\,\phi^2, & B_9 \equiv (\Delta\partial_i\phi)(\Delta\phi)(\partial_i\phi)\,\phi, \\ B_{10} \equiv (\Delta\partial_i\phi)(\partial_i\partial_j\phi)(\partial_j\phi)\,\phi, & B_{11} \equiv (\partial_i\partial_j\partial_k\phi)(\partial_i\partial_j\phi)(\partial_k\phi)\,\phi, & B_{12} \equiv (\Delta\phi)(\Delta\phi)(\Delta\phi)\,\phi, \\ B_{13} \equiv (\Delta\phi)(\partial_i\partial_j\phi)(\partial_i\partial_j\phi)\,\phi, & B_{14} \equiv (\partial_i\partial_j\phi)(\partial_j\partial_k\phi)(\partial_k\partial_i\phi)\,\phi, & B_{15} \equiv (\Delta\partial_i\phi)(\partial_i\phi)(\partial_j\phi)(\partial_j\phi)\,, \\ B_{16} \equiv (\partial_i\partial_j\partial_k\phi)(\partial_i\phi)(\partial_j\phi)(\partial_k\phi)\,, & B_{17} \equiv (\Delta\phi)(\Delta\phi)(\partial_i\phi)(\partial_i\phi)\,, & B_{18} \equiv (\Delta\phi)(\partial_i\partial_j\phi)(\partial_i\phi)(\partial_j\phi)\,, \\ B_{19} \equiv (\partial_i\partial_j\phi)(\partial_i\partial_j\phi)(\partial_k\phi)(\partial_k\phi)\,, & B_{20} \equiv (\partial_i\partial_j\phi)(\partial_i\partial_k\phi)(\partial_j\phi)(\partial_k\phi)\,. & \end{array} \tag{A4}$$

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[4] In [20], non-Gaussianity of cosmological perturbations in Hořava-Lifshitz gravity is discussed. However, in that paper, non-renormalizable interaction terms are considered as the main source of non-Gaussianity. In the present paper, on the other hand, all terms in the action are power-counting renormalizable and we consider marginal ones as the main source of non-Gaussianity.

Apparently, there are sixteen total derivatives as follows,

$$
\begin{aligned}
&\partial_i\left[\left(\Delta^2\partial_i\phi\right)\phi^3\right]=B_1+3B_2, && \partial_i\left[\left(\Delta^2\phi\right)\left(\partial_i\phi\right)\phi^2\right]=B_2+B_3+2B_5,\\
&\partial_i\left[\left(\Delta\partial_i\partial_j\phi\right)\left(\partial_j\phi\right)\phi^2\right]=B_2+B_4+2B_6, && \partial_i\left[\left(\Delta\partial_i\phi\right)\left(\Delta\phi\right)\phi^2\right]=B_3+B_7+2B_9,\\
&\partial_i\left[\left(\Delta\partial_j\phi\right)\left(\partial_i\partial_j\phi\right)\phi^2\right]=B_4+B_7+2B_{10}, && \partial_i\left[\left(\partial_i\partial_j\partial_k\phi\right)\left(\partial_j\partial_k\phi\right)\phi^2\right]=B_4+B_8+2B_{11},\\
&\partial_i\left[\left(\Delta\partial_i\phi\right)\left(\partial_j\phi\right)\left(\partial_j\phi\right)\phi\right]=B_5+2B_{10}+B_{15}, && \partial_i\left[\left(\Delta\partial_j\phi\right)\left(\partial_i\phi\right)\left(\partial_j\phi\right)\phi\right]=B_6+B_9+B_{10}+B_{15},\\
&\partial_i\left[\left(\partial_i\partial_j\partial_k\phi\right)\left(\partial_j\phi\right)\left(\partial_k\phi\right)\phi\right]=B_6+2B_{11}+B_{16}, && \partial_i\left[\left(\Delta\phi\right)\left(\Delta\phi\right)\left(\partial_i\phi\right)\phi\right]=2B_9+B_{12}+B_{17},\\
&\partial_i\left[\left(\Delta\phi\right)\left(\partial_i\partial_j\phi\right)\left(\partial_j\phi\right)\phi\right]=B_9+B_{10}+B_{13}+B_{18}, && \partial_i\left[\left(\partial_i\partial_j\phi\right)\left(\partial_j\partial_k\phi\right)\left(\partial_k\phi\right)\phi\right]=B_{10}+B_{11}+B_{14}+B_{20},\\
&\partial_i\left[\left(\partial_j\partial_k\phi\right)\left(\partial_j\partial_k\phi\right)\left(\partial_i\phi\right)\phi\right]=2B_{11}+B_{13}+B_{19}, && \partial_i\left[\left(\Delta\phi\right)\left(\partial_i\phi\right)\left(\partial_j\phi\right)\left(\partial_j\phi\right)\right]=B_{15}+B_{17}+2B_{18},\\
&\partial_i\left[\left(\partial_j\partial_k\phi\right)\left(\partial_i\phi\right)\left(\partial_j\phi\right)\left(\partial_k\phi\right)\right]=B_{16}+B_{18}+2B_{20}, && \partial_i\left[\left(\partial_i\partial_j\phi\right)\left(\partial_j\phi\right)\left(\partial_k\phi\right)\left(\partial_k\phi\right)\right]=B_{15}+B_{19}+2B_{20}.
\end{aligned}
\tag{A5}
$$

Two of them are not independent. Thus, marginal quartic terms in the action can be written as a linear combination of six independent terms. In this paper, we use the following terms,

$$
\begin{aligned}
S_4=\int dtd^3x\,\frac{1}{M^6a(t)^3}\big\{&\beta_1\phi^3\Delta^3\phi+\beta_2\phi^2(\Delta\phi)(\Delta^2\phi)+\beta_3\phi(\Delta\phi)^3\\
&+\beta_4\phi^2(\Delta\partial_i\phi)^2+\beta_5\phi^2(\partial_i\partial_j\partial_k\phi)^2+\beta_6(\partial_i\partial_j\partial_k\phi)(\partial_i\phi)(\partial_j\phi)(\partial_k\phi)\big\}.
\end{aligned}
\tag{A6}
$$

With the shift symmetry, we have to consider only the last six terms in (A4) and only the last three combinations in (A5). Therefore, the number of the independent terms is three. In the notation (A6), the shift symmetry imposes the constraints as

$$
\beta_1=\frac{1}{12}\beta_5,\qquad \beta_2=\beta_4+\frac{3}{4}\beta_5,\qquad \beta_3=-\beta_4-\frac{3}{2}\beta_5,
\tag{A7}
$$

and then the action can be transformed into the following form,

$$
S_4=\int dtd^3x\,\frac{1}{M^6a(t)^3}\big\{\left(\beta_4+2\beta_5\right)\left(\Delta\phi\right)^2\left(\partial_i\phi\right)^2+\beta_5\left(\partial_i\partial_j\phi\right)^2\left(\partial_k\phi\right)^2+\beta_6(\partial_i\partial_j\partial_k\phi)(\partial_i\phi)(\partial_j\phi)(\partial_k\phi)\big\}.
\tag{A8}
$$

This manifestly has the shift symmetry.

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[1] P. Horava, Phys. Rev. D **79**, 084008 (2009) [arXiv:0901.3775 [hep-th]].
P. Horava, Phys. Rev. Lett. **102**, 161301 (2009) [arXiv:0902.3657 [hep-th]].
[2] S. Mukohyama, JCAP **0906**, 001 (2009) [arXiv:0904.2190 [hep-th]].
[3] G. Calcagni, JHEP **0909**, 112 (2009) [arXiv:0904.0829 [hep-th]].
R. Brandenberger, Phys. Rev. D **80**, 043516 (2009) [arXiv:0904.2835 [hep-th]].
E. Kiritsis and G. Kofinas, Nucl. Phys. B **821**, 467 (2009) [arXiv:0904.1334 [hep-th]].
S. Maeda, S. Mukohyama and T. Shiromizu, Phys. Rev. D **80**, 123538 (2009) [arXiv:0909.2149 [astro-ph.CO]].
S. Mukohyama, K. Nakayama, F. Takahashi and S. Yokoyama, Phys. Lett. B **679**, 6 (2009) [arXiv:0905.0055 [hep-th]].
T. Takahashi and J. Soda, Phys. Rev. Lett. **102**, 231301 (2009) [arXiv:0904.0554 [hep-th]].
S. Mukohyama, Phys. Rev. D **80**, 064005 (2009) [arXiv:0905.3563 [hep-th]].
S. Mukohyama, JCAP **0909**, 005 (2009) [arXiv:0906.5069 [hep-th]].
M. Henneaux, A. Kleinschmidt and G. L. Gomez, Phys. Rev. D **81**, 064002 (2010) [arXiv:0912.0399 [hep-th]].
C. Charmousis, G. Niz, A. Padilla and P. M. Saffin, JHEP **0908**, 070 (2009) [arXiv:0905.2579 [hep-th]].
A. Wang and R. Maartens, Phys. Rev. D **81**, 024009 (2010) [arXiv:0907.1748 [hep-th]].
D. Blas, O. Pujolas and S. Sibiryakov, Phys. Rev. Lett. **104**, 181302 (2010) [arXiv:0909.3525 [hep-th]].
K. Koyama and F. Arroja, JHEP **1003**, 061 (2010) [arXiv:0910.1998 [hep-th]].
K. Izumi and S. Mukohyama, Phys. Rev. D **81**, 044008 (2010) [arXiv:0911.1814 [hep-th]].
C. Doran, Phys. Rev. D **61**, 067503 (2000) [arXiv:gr-qc/9910099].
K. i. Maeda, Y. Misonoh and T. Kobayashi, arXiv:1006.2739 [hep-th].
T. Kobayashi, Y. Urakawa and M. Yamaguchi, JCAP **0911**, 015 (2009) [arXiv:0908.1005 [astro-ph.CO]].
T. Harada, U. Miyamoto and N. Tsukamoto, arXiv:0911.1187 [gr-qc].
[4] D. H. Lyth and D. Wands, Phys. Lett. B **524**, 5 (2002) [arXiv:hep-ph/0110002].
K. Enqvist and M. S. Sloth, Nucl. Phys. B **626**, 395 (2002) [arXiv:hep-ph/0109214].
T. Moroi and T. Takahashi, Phys. Lett. B **522**, 215 (2001) [Erratum-ibid. B **539**, 303 (2002)] [arXiv:hep-ph/0110096].
[5] G. Dvali, A. Gruzinov and M. Zaldarriaga, Phys. Rev. D **69**, 023505 (2004) [arXiv:astro-ph/0303591].
L. Kofman, arXiv:astro-ph/0303614.
[6] E. Komatsu *et al.*, arXiv:1001.4538 [astro-ph.CO].

[7] J. M. Maldacena, JHEP **0305**, 013 (2003) [arXiv:astro-ph/0210603].
[8] A. D. Linde and V. F. Mukhanov, Phys. Rev. D **56**, 535 (1997) [arXiv:astro-ph/9610219].
N. Bartolo, S. Matarrese and A. Riotto, Phys. Rev. D **65**, 103505 (2002) [arXiv:hep-ph/0112261].
F. Bernardeau and J. P. Uzan, Phys. Rev. D **66**, 103506 (2002) [arXiv:hep-ph/0207295].
F. Bernardeau and J. P. Uzan, Phys. Rev. D **67**, 121301 (2003) [arXiv:astro-ph/0209330].
G. Dvali, A. Gruzinov and M. Zaldarriaga, Phys. Rev. D **69**, 023505 (2004) [arXiv:astro-ph/0303591].
N. Bartolo, E. Komatsu, S. Matarrese and A. Riotto, Phys. Rept. **402**, 103 (2004) [arXiv:astro-ph/0406398].
D. Seery and J. E. Lidsey, JCAP **0506**, 003 (2005) [arXiv:astro-ph/0503692].
D. Seery and J. E. Lidsey, JCAP **0509**, 011 (2005) [arXiv:astro-ph/0506056].
M. Sasaki, J. Valiviita and D. Wands, Phys. Rev. D **74**, 103003 (2006) [arXiv:astro-ph/0607627].
X. Chen, M. x. Huang, S. Kachru and G. Shiu, JCAP **0701**, 002 (2007) [arXiv:hep-th/0605045].
T. Suyama and M. Yamaguchi, Phys. Rev. D **77**, 023505 (2008) [arXiv:0709.2545 [astro-ph]].
F. Arroja and K. Koyama, Phys. Rev. D **77**, 083517 (2008) [arXiv:0802.1167 [hep-th]].
F. Arroja, S. Mizuno and K. Koyama, JCAP **0808**, 015 (2008) [arXiv:0806.0619 [astro-ph]].
M. Sasaki, Prog. Theor. Phys. **120**, 159 (2008) [arXiv:0805.0974 [astro-ph]].
A. Naruko and M. Sasaki, Prog. Theor. Phys. **121**, 193 (2009) [arXiv:0807.0180 [astro-ph]].
T. Suyama and F. Takahashi, JCAP **0809**, 007 (2008) [arXiv:0804.0425 [astro-ph]].
X. Gao, JCAP **0806**, 029 (2008) [arXiv:0804.1055 [astro-ph]].
H. R. S. Cogollo, Y. Rodriguez and C. A. Valenzuela-Toledo, JCAP **0808**, 029 (2008) [arXiv:0806.1546 [astro-ph]].
K. Ichikawa, T. Suyama, T. Takahashi and M. Yamaguchi, Phys. Rev. D **78**, 023513 (2008) [arXiv:0802.4138 [astro-ph]].
C. T. Byrnes, JCAP **0901**, 011 (2009) [arXiv:0810.3913 [astro-ph]].
D. Langlois, F. Vernizzi and D. Wands, JCAP **0812**, 004 (2008) [arXiv:0809.4646 [astro-ph]].
C. Hikage, K. Koyama, T. Matsubara, T. Takahashi and M. Yamaguchi, Mon. Not. Roy. Astron. Soc. **398**, 2188 (2009) [arXiv:0812.3500 [astro-ph]].
M. Kawasaki, K. Nakayama, T. Sekiguchi, T. Suyama and F. Takahashi, JCAP **0811**, 019 (2008) [arXiv:0808.0009 [astro-ph]].
Q. G. Huang, JCAP **0811**, 005 (2008) [arXiv:0808.1793 [hep-th]].
Q. G. Huang, JCAP **0905**, 005 (2009) [arXiv:0903.1542 [hep-th]].
Q. G. Huang, JCAP **0906**, 035 (2009) [arXiv:0904.2649 [hep-th]].
C. T. Byrnes and G. Tasinato, JCAP **0908**, 016 (2009) [arXiv:0906.0767 [astro-ph.CO]].
C. Cheung, A. L. Fitzpatrick, J. Kaplan and L. Senatore, JCAP **0802**, 021 (2008) [arXiv:0709.0295 [hep-th]].
X. Chen, M. x. Huang, S. Kachru and G. Shiu, JCAP **0701**, 002 (2007) [arXiv:hep-th/0605045].
[9] M. Alishahiha, E. Silverstein and D. Tong, Phys. Rev. D **70**, 123505 (2004) [arXiv:hep-th/0404084].
D. Langlois, S. Renaux-Petel, D. A. Steer and T. Tanaka, Phys. Rev. Lett. **101**, 061301 (2008) [arXiv:0804.3139 [hep-th]].
D. Langlois, S. Renaux-Petel, D. A. Steer and T. Tanaka, Phys. Rev. D **78**, 063523 (2008) [arXiv:0806.0336 [hep-th]].
S. Li, Y. F. Cai and Y. S. Piao, Phys. Lett. B **671**, 423 (2009) [arXiv:0806.2363 [hep-ph]].
X. Gao and B. Hu, JCAP **0908**, 012 (2009) [arXiv:0903.1920 [astro-ph.CO]].
Y. F. Cai and H. Y. Xia, Phys. Lett. B **677**, 226 (2009) [arXiv:0904.0062 [hep-th]].
D. Langlois, S. Renaux-Petel and D. A. Steer, JCAP **0904**, 021 (2009) [arXiv:0902.2941 [hep-th]].
J. Khoury and F. Piazza, JCAP **0907**, 026 (2009) [arXiv:0811.3633 [hep-th]].
S. Mizuno, F. Arroja, K. Koyama and T. Tanaka, Phys. Rev. D **80**, 023530 (2009) [arXiv:0905.4557 [hep-th]].
M. x. Huang, G. Shiu and B. Underwood, Phys. Rev. D **77**, 023511 (2008) [arXiv:0709.3299 [hep-th]].
N. Arkani-Hamed, P. Creminelli, S. Mukohyama and M. Zaldarriaga, JCAP **0404**, 001 (2004) [arXiv:hep-th/0312100].
R. Holman and A. J. Tolley, JCAP **0805**, 001 (2008) [arXiv:0710.1302 [hep-th]].
[10] D. Seery and J. E. Lidsey, JCAP **0701**, 008 (2007) [arXiv:astro-ph/0611034].
D. Seery, M. S. Sloth and F. Vernizzi, JCAP **0903**, 018 (2009) [arXiv:0811.3934 [astro-ph]].
Y. Rodriguez and C. A. Valenzuela-Toledo, Phys. Rev. D **81**, 023531 (2010) [arXiv:0811.4092 [astro-ph]].
X. Chen, B. Hu, M. x. Huang, G. Shiu and Y. Wang, JCAP **0908**, 008 (2009) [arXiv:0905.3494 [astro-ph.CO]].
X. Gao, M. Li and C. Lin, JCAP **0911**, 007 (2009) [arXiv:0906.1345 [astro-ph.CO]].
X. Chen, B. Hu, M. x. Huang, G. Shiu and Y. Wang, JCAP **0908**, 008 (2009) [arXiv:0905.3494 [astro-ph.CO]].
X. Chen, M. x. Huang and G. Shiu, Phys. Rev. D **74**, 121301 (2006) [arXiv:hep-th/0610235].
F. Arroja, S. Mizuno, K. Koyama and T. Tanaka, Phys. Rev. D **80**, 043527 (2009) [arXiv:0905.3641 [hep-th]].
S. Mizuno, F. Arroja and K. Koyama, Phys. Rev. D **80**, 083517 (2009) [arXiv:0907.2439 [hep-th]].
S. Renaux-Petel, JCAP **0910**, 012 (2009) [arXiv:0907.2476 [hep-th]].
K. Izumi and S. Mukohyama, JCAP **1006**, 016 (2010) [arXiv:1004.1776 [hep-th]].
Q. G. Huang, JCAP **1007**, 025 (2010) [arXiv:1004.0808 [astro-ph.CO]].
D. M. Regan, E. P. S. Shellard and J. R. Fergusson, Phys. Rev. D **82**, 023520 (2010) [arXiv:1004.2915 [astro-ph.CO]].
N. Bartolo, M. Fasiello, S. Matarrese and A. Riotto, arXiv:1006.5411 [astro-ph.CO].
S. Mizuno and K. Koyama, arXiv:1007.1462 [hep-th].
[11] S. Weinberg, Phys. Rev. D **72**, 043514 (2005) [arXiv:hep-th/0506236].
[12] E. Komatsu and D. N. Spergel, Phys. Rev. D **63**, 063002 (2001) [arXiv:astro-ph/0005036].
[13] L. Senatore, K. M. Smith and M. Zaldarriaga, JCAP **1001**, 028 (2010) [arXiv:0905.3746 [astro-ph.CO]].
[14] P. Creminelli, A. Nicolis, L. Senatore, M. Tegmark and M. Zaldarriaga, JCAP **0605**, 004 (2006) [arXiv:astro-ph/0509029].
[15] D. Babich, P. Creminelli and M. Zaldarriaga, JCAP **0408**, 009 (2004) [arXiv:astro-ph/0405356].

[16] C. Cheung, P. Creminelli, A. L. Fitzpatrick, J. Kaplan and L. Senatore, JHEP **0803**, 014 (2008) [arXiv:0709.0293 [hep-th]].
[17] B. Chen, S. Pi and J. Z. Tang, JCAP **0908**, 007 (2009) [arXiv:0905.2300 [hep-th]].
[18] S. Mukohyama, arXiv:1007.5199 [hep-th].
[19] P. Horava and C. M. Melby-Thompson, arXiv:1007.2410 [hep-th].
[20] X. Gao, arXiv:0904.4187 [hep-th].